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RESONANT PERIODIC ORBITS IN SUN-JUPITER AND SUN-MERCURY PHOTOGRAVITATIONAL RESTRICTED THREE-BODY PROBLEM

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ABSTRACT

We explore the effect of radiation pressure of the Sun on the resonant periodic orbits in the Sun-Jupiter and Sun-Mercury systems, in the frame work of planar circular restricted three body problem, with the help of Poincaré surface of section. First order and and second order interior resonances are located. The effect of radiation pressure is studied on the location and size of periodic orbits. By increasing the radiation pressure, we have located the first merging and second merging points of first order interior resonance periodic orbits for different values of Jacobian constant C. At the time of merging, these orbits become near-circular. The period and size of these orbits reduce with the increase in radiation pressure.

KEYWORDS: Restricted Three Body Problem, Interior Resonance, Poincaré Surface of Section & Solar Radiation Pressure

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Original Article

1. INTRODUCTION

The restricted three-body problem (RTBP) has beenextensively studied for the dynamics of a secondary body of negligible mass, moving under the gravitational attraction of the two primaries. The RTBP has many applications in the solar system dynamics, and several modifications of this problem have been proposed for the study of the motion of a masslessparticle. The book of Szebehely [1] provides systematic coverage of the literature on the subject as well as derivations of some of the important results. These modifications include some of the perturbing forces such as oblateness, variations of the masses of the primaries and radiation pressure of the primaries. In this paper, we study the effect of radiation pressure on the interior resonance periodic orbits.

The photogravitational restricted three-body problem takes into account only the radiation pressure component of the radiation drag, which is the next most powerful component after the gravitational forces. Poynting [2], and Robertson [3] showed that the effect produced by the radiation force on the dynamics of small body depend on its particular geometry, physical and physicochemical characteristics. Radzievskii[4] proposed a simplified theory, and since then some of the notable research in the photo-gravitational restricted three body problem are by Chernikov [5], Perezhogin[6], Bhatnagar and Chawla[7] Schuerman[8] Simmons et al. [9] Roman [10] Kushvah and Ishwar[11] and Das et al. [12]

A number of iterative schemes to compute periodic orbits numerically have been used in the literature. Some of them are by Broucke[13] andBruno[14]. The set of stable periodic and quasi-periodic trajectories define regions of regular motion or stability “islands” that spread in a chaotic “sea” made of trajectories with high sensitivity, with respect to the initial condition. PSS gives a qualitative picture of stability regions in the planar

problems. Winter [15]studied the location and stability of periodic and quasi-periodic orbits in the Earth-Moon system. Dutt and Sharma [16]employed PSS method to identify periodic, quasi-periodic solutions and chaotic regions for Earth-Moon system and Sun-Mars system. Beevi and Sharma [17] considered the oblateness of the more massive primary (Saturn) and studied the periodic and quasi-periodic orbits in the Saturn-Titan system.

The planetary data in recent years has provided striking confirmation that our solar system is a highly structured assembly of orbiting bodies, and the subtle gravitational effect that determines its dynamical structure is the phenomenon of resonance. The ring system of Uranus has a number of examples of resonant phenomena. For example, Rosalind and Cordelia are close to 5:3 resonance. In the context of orbital evolution through resonance, useful reviews have been given by Greenberg [18], Peale [19]. Asteroids residing in the first-order mean motion resonances with Jupiter hold important information about the processes that set the final architecture of giant planets Brož and Vokrouhlicky [20]. It is known that the population of the asteroids exist in the Jovian first-order mean motion resonances 2:1(Hecuba-gap group), 3:2 (Hilda group) and 4:3(Thule group). The authors main results were an update of the observed 2:1, 3:2 and 4:3 resonant populations; discovery of two new objects in the 4:3 resonance and description of two asteroid families located inside the 3:2 group. Quarle, et al[21]have identify and classified the mean-motion resonance for the coplanar CRTBP for different mass ratio, and recently Voyatzis et al. [22]studied inclined asymmetric periodic solutions in exterior mean-motion resonances in the Sun–Neptune–trans-Neptunian object system.

The periodic orbits around the Sun with 5:3,4:3, 3:2 and 2:1 first-order interior resonances in the framework of Sun-Jupiter photo gravitational restricted three-body problem (PRTBP) are studied. We have carried out a detailed study in PRTBP to find the effect of solar radiation pressure (ε) and Jacobian constant C on these obits in the Sun-Jupiter and Sun-Mercury RTBP.

2. PLANAR CIRCULAR RESTRICTED THREE-BODY PROBLEM

Let m_1 and m_2 $(m_1 > m_2)$ be two masses of first and second primary bodies, respectively. These two masses have circular orbits about their common center of mass in a plane under the influence of their mutual gravitational attraction. In terms of dimensionless synodic coordinate system with origin at the center of mass of the two bodies, whose locations are at (μ,0) and (μ-1,0) with equatorial plane same as the plane of motion, where $\mu = m_1/(m_1+m_2) \leq 1/2$, (=0.0009537284) m_1 and m_2 are masses of the Sun and Jupiter, respectively. The motion of an infinite small particle under the gravitational attraction of two primaries is simulated by numerically integrating the planer circular restricted three-body problem.

The effect of radiation pressure of a source can be expressed by a mass reduction factor q =1 - ε, where the radiation coefficient ε is the ratio of the force F_p which is caused by radiation to the force F_g which results from gravitation, i.e., $\varepsilon = F_p/F_g$. q is expressed in terms of particle radius 'a', density 'δ' and radiation pressure efficiency 'χ' (in CGS system) as

$$q = 1 - \frac{5.6 \times 10^{-5}}{a\delta}\chi \tag{2.1}$$

Knowing the mass and the luminosity of the radiating body, ε can be found for any given radius and density. Solar radiation pressure force F_p changes with distance by the same law of gravitational attraction force F_g and acts opposite to it. Thus, Sun's resulting force acting on the particle is Sharma [23]; Kalvouridis et al. [24]

$$F = F_g - F_p = (1 - F_p/F_g)\, F_g = (q)F_g \tag{2.2}$$

For q =1, there is no radiation effect, and for $0 < q \leq 1$, gravitational force exceeds radiation and we consider this case for our detailed study.

The planar equations of motion of the third body are (Bhatnagar and Chawla [25])

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x}, \tag{2.3}$$

$$\ddot{y} + 2\,\dot{x} = \frac{\partial \Omega}{\partial y}, \tag{2.4}$$

where

$$\Omega = \frac{1}{2}[(1-\mu)r_1{}^2 + \mu r_2^2] + \frac{q\,(1-\mu)}{r_1} + \frac{\mu}{r_2} \tag{2.5}$$

$$r_1^2 = (x-\mu)^2 + y^2,$$

$$r_2^2 = (x+1-\mu)^2 + y^2,$$

The Jacobi integral is

$$\dot{x}^2 + \dot{y}^2 = 2\,\Omega - C. \tag{2.6}$$

3. NUMERICAL RESULTS

The families of periodic orbits around the Sun in the Sun Jupiter system are studied using the Poincaré Surface of section method. We have constructed the PSS in the x, $\dot{x}$ plane. By defining the plane, say y=0, in resulting three-dimensional space, the values of x and $\dot{x}$are plotted every time the particle has y=0, whenever trajectory intersects the plane in a particular direction, say $\dot{y} > 0$. Poincare surface of section technique is good at determining the regular or chaotic nature of the trajectory. If there are smooth, well defined islands, then the trajectory is likely to be regular and the islands correspond to oscillation around a period orbit. As the curve shrink down to a point, the point represents a periodic orbit as par Kolmogorov- Arnold –Moser (KAM) theory. Any fuzzy distribution of points in the surface of section implies that the trajectory is chaotic.

The starting conditions for numerical integration were chosen as follows: for each value of Jacobian constant C, the value of x was selected so that $y = \dot{x} = 0$ and $\dot{y} > 0$. In order to generate the Poincaré surface of section, the equation of motion (2.3) and (2.4) are integrated using fourth-order Runge-Kutta-Gill method with integration step size Δt of 0.0005. To obtain periodic orbit in Sun-Jupiter system, we have generated PSS for various values of the Jacobi constant C, with 0.0009537284.

4. RESONANCE LOCATIONS

In the present study, we have generated the trajectories of the third particlefor different initial conditions for the Sun-Jupiter system by considering Sun as source of radiation. We have generated the trajectories at various locations for C = 2.9 and $\varepsilon = 0$ given in Figure 1. We consider the trajectories at various locations to study the order of resonance. The trajecory of the test particle with starting value x_0=0.1635, y_0 = 0.0, $\dot{x}_0$ = 0.0 and $\dot{y}_0 > 0.0$ is determined from (2.6) and this trajectory is close to 5:3 second order resonance Murray and Dermott [26]. The trajectory forC = 2.9 at x = 0.2743 is close to 2:1 first order, interior resonance. It is observed that the trajectory for C = 2.9 at x = 0.4460 is close to 3:2 first order, interior resonance. If we consider the trajectory at C = 2.9, x = 0.4318, it is of 4:3 first order interior resonance.

To study the effect of radiation pressure of Sun, we consider the orbit having resonance 5:3 as shown in Figure 2 for C = 2.9 and ε = 1- q = 1- 0.99 = 0.01 at x = 0.1703. The transformation of this periodic orbit with 5:3 interior resonance is shown in Figures 3 to 6 by increasing the radiation pressure ε from 0.11 to 0.20. It is also observed that the eccentricity of these orbits decreases with the increase in radiation pressure ε. Figure 7 provides the initial locations of 5:3 periodic orbits for Ɛ = 0.0 to 0.15. It may be noted that with the increase in radiation pressure ε, the orbits move towards Jupiter.

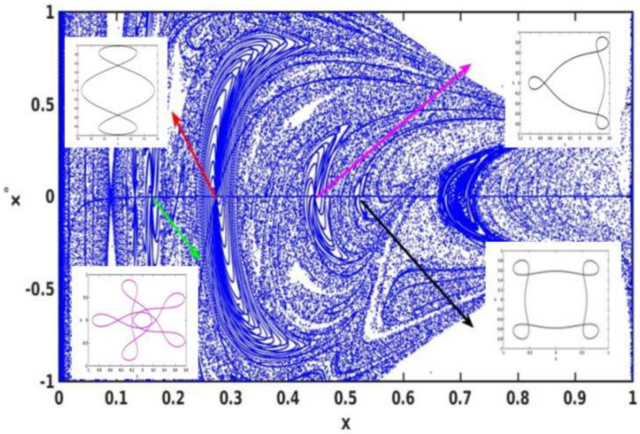

Figure 1: PSS for Jacobi Constant C=2.9 and ε= 0

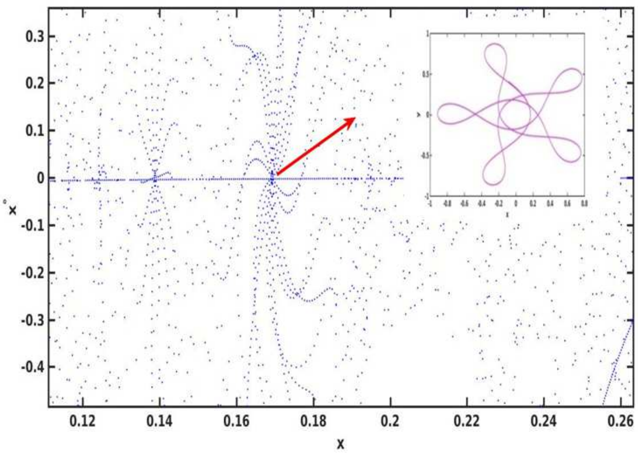

Figure 2: PSS for Jacobi Constant C=2.9, ε=0.01

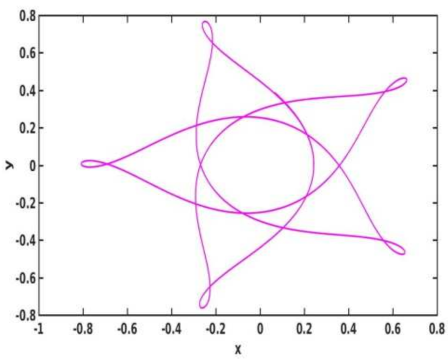

Figure 3: C=2.95, ε= 0.11, x= 0.2432

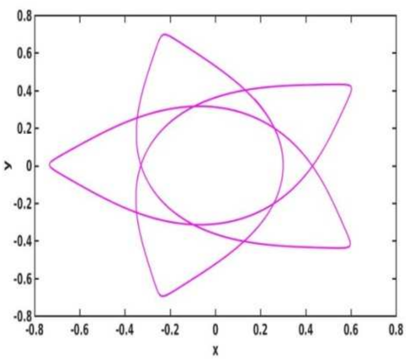

Figure 4: C=2.95, ε=0.13, x= 0.3003

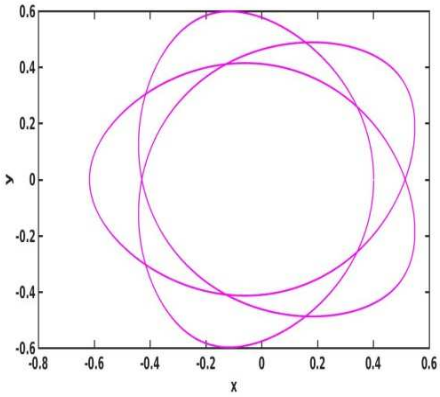

Figure 5: C=2.95, ε=0.13, x= 0.401

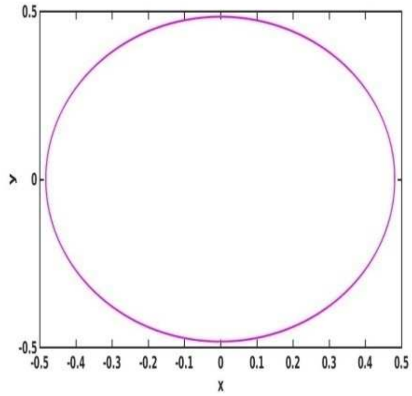

Figure 6: C=2.95, ε=0.20, x= 0.4817

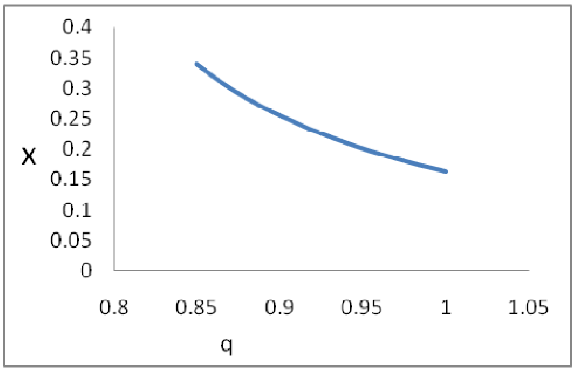

Figure 7: Initial Location of 5:3 Periodic Orbits as a Function of Radiation Pressure q

5. MERGING OF RESONANT PERIODIC ORBITS 4:3; 3:2 AND 2:1

The Poincaré surface of section (PSS) method is used to find 2:1, 3:2 and 4:3interior resonance periodic orbits around the Sun, under the effect of the radiation pressure. The distance between the two consecutive starting conditions Δx and time step Δt in the numerical integration were suitably selected. PSS were generated for Jacobi constant C = 2.8 without the radiation pressure and presented in Figure 8. These orbits starting between x = 0.2 and x = 0.45 are shown in Figure 8. They also shift towards Jupiter with increase in radiation pressure (ε). Their shape changes gradually to elliptic orbits. It is observed that with increase in ε, 4:3 resonant periodic orbit merges with 3:2 resonant periodic orbit at x = 0.7016 and the resonance of the merged periodic orbit becomes 1:1. This orbit shifts towards Sun with further increase in radiation pressure. As ε increases from 0.13 to 0.17, the periodic orbit with 2:1 interior resonance shift towards Jupiter and merges with the previous one at x = 0.5315 and become near 1:1 resonance. Table 1 provides a comparison of the time period of the periodic orbits for ε = 0 to 0.17. It may be seen from Figures 9 and 10 that as ε increases from 0.13 to 0.17, the orbit becomes gradually circular and smaller in size.

We have also obtained PSS for Sun-Jupiter RTBP for C = 3.0. In this case, 4:3 resonant periodic orbits merge with 3:2 resonant periodic orbit at x = 0.7237 and the resonance of the merged periodic orbit becomes 1:1. This orbit shifts towards Sun with further increase in radiation pressure. As ε increases from 0.03 to 0.09, the periodic orbit with 2:1 interior resonance shift towards Jupiter and merges with the previous one at x = 0.5824 and become near 1:1 resonance. Table 2 provides a comparison of the time period of the periodic orbits for ε = 0 to 0.09.

We have plotted the values of ε vs. C for the first merging between 3:2 and 4:3 resonant orbits. It is noticed that, as C increases from 0.2 to 3.1, radiation pressure coefficient ε decreases. Figure 13 provides values of radiation pressure (ε) for different values of Jacobian constant C from 0.2 to 3.1, where 4:3 resonant periodic orbit merges with 3:2 resonant periodic orbit to become near 1:1 resonance orbit. Figure 14provides values of radiation pressure (ε) for different values of Jacobian constant C from 0.2 to 3.1, where 2:1 resonant periodic orbit merges with 1:1resonant periodic orbit to become near 1:1 resonance orbit.

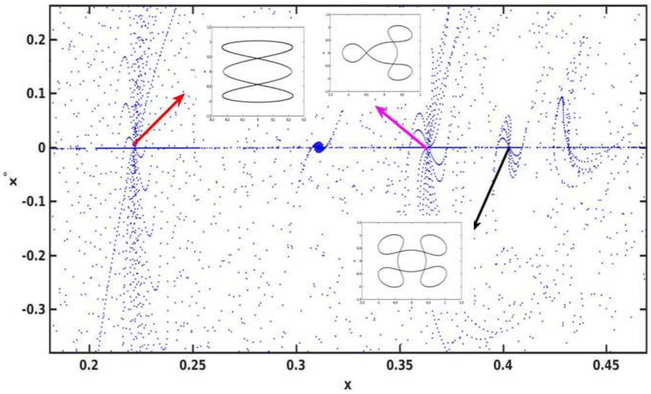

Figure 8: PSS for Jacobi Constant C=2.8 and ε= 0

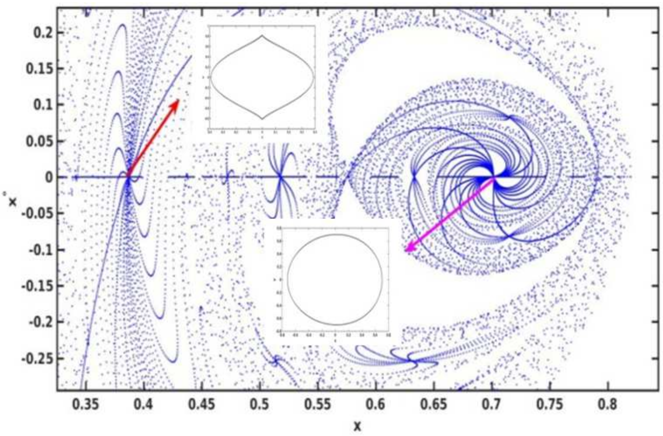

Figure 9: PSS for Jacobi Constant C=2.8 and ε= 0.13

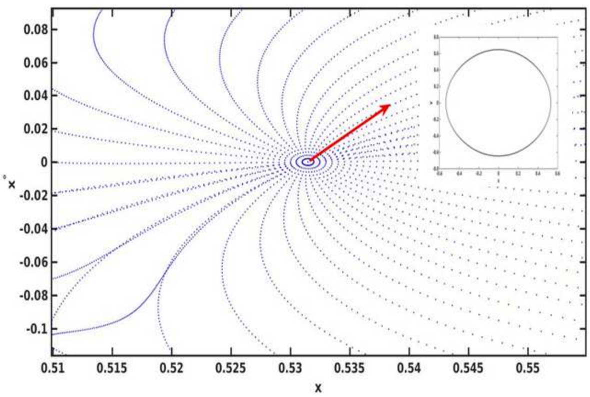

Figure 10: PSS for Jacobi Constant C=2.8 and ε= 0.17

Table 1: Initial Location and Time Period of the Orbit

ε	Resonance					
	2:1		3:2		4:3	
	Location	Period	Location	Period	Location	Period
0	0.2200	6.2793	0.3639	12.5656	0.4318	18.8986
0.01	0.2303	6.2792	0.3764	12.5661	0.4450	18.8898
0.04	0.2578	6.2791	0.4193	12.5624	0.4930	18.8686
0.07	0.2908	6.2774	0.4745	12.5557	0.5589	18.8479
0.10	0.3320	6.2770	0.5547	12.5340	0.6690	18.7635
0.13	0.3870	6.2711	0.7016	10.9960	**First Merge**	
0.16	0.4761	6.2500	0.6262	7.1032		
0.17	0.5315	6.2117	**Second Merge**			

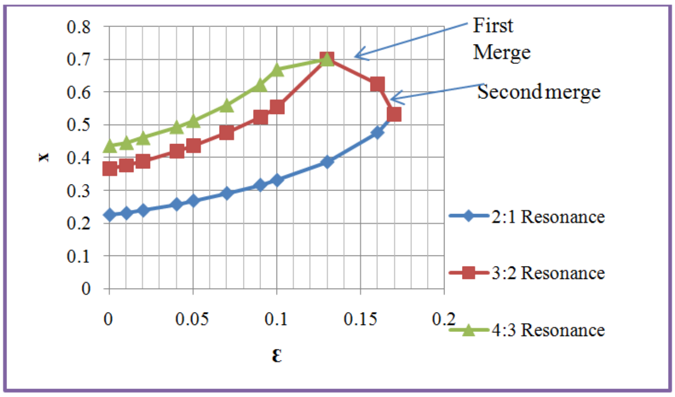

Figure 11: First and Second Merging Location for C=2.8

Table 2: Initial Location and Time Period of the Orbits

ε	Resonance					
	2:1		3:2		4:3	
	Location	Period	Location	Period	Location	Period
0	0.3411	6.2762	0.5683	12.5451	0.6830	18.771
0.01	0.3560	6.2750	0.6032	12.5181	0.7449	18.583
0.02	0.3724	6.2741	0.6504	12.4725	0.8036	15.513
0.03	0.3907	6.2659	0.7237	11.9886	**First Merge**	
0.04	0.411	6.2657	0.7145	10.1296		
0.05	0.4347	6.2653	0.6885	8.7694		
0.07	0.4996	6.2651	0.6460	6.9864		
0.09	0.5230	6.2510	**Second Merge**			

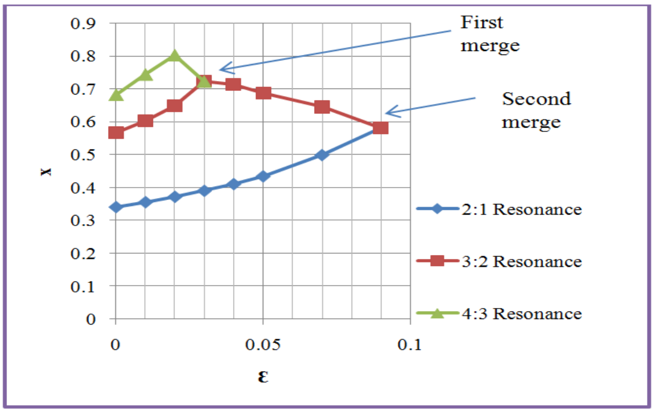

Figure 12: First and Second Merging Location for C=3.0

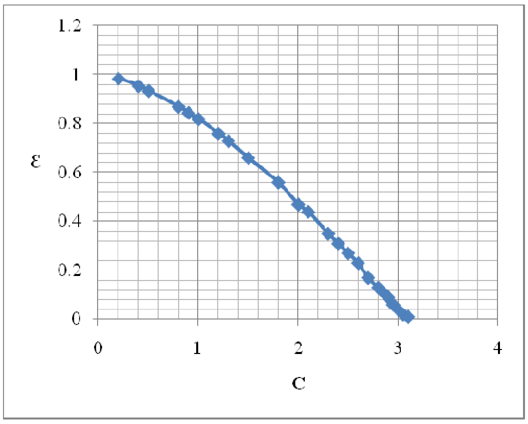

Figure 13: First Merge

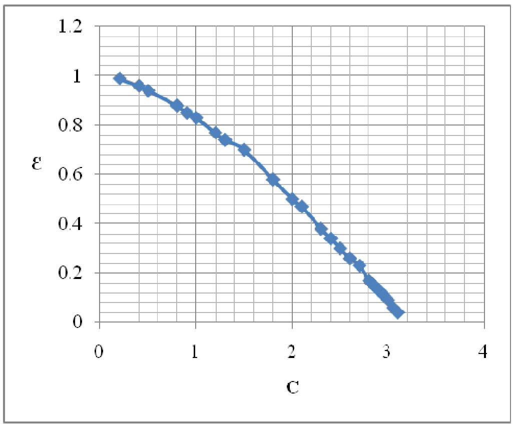

Figure 14: Second Merge

We have studied the effect of radiation pressure of Sun on the size of the periodic orbits for C = 2.99, on the merged resorant orbits (4:3, 3.2 and 2:1into 1:1). Figure 15 provides PSS for C = 2.99 when q = 0.83, 0.85, 0.87, 0.89, 0.91. It is seen that the size of the periodic orbit decrease as radiation pressure increase.

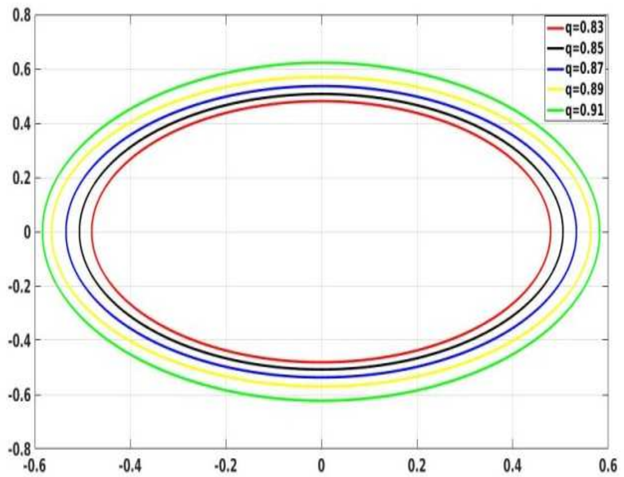

Figure 15: Sun –Centered Periodic Orbits for C=2.99

6. EVOLUTION OF PERIODIC ORBIT IN SUN-MERCURY SYSTEM

To study the effect of the mass parameter, we have obtained PSS for the Sun-Mercury system, whose mass ratio μ= 0.000000165956, is very small for one value of C=2.9. In this case, 4:3 resonant periodic orbit merges with 3:2 resonant periodic orbit at x = 0.695, and the resonance of the merged periodic orbit becomes 1:1. This orbit shifts towards Sun with further increase in radiation pressure. As ε increases from 0.09 to 0.13, the periodic orbit with 2:1 interior resonance shifts towards Jupiter and merges with the previous one at x = 0.596 and become near 1:1 resonance. It is interesting to note that both the mergings are found at the same location, as obtained in the Sun-Jupiter system. However, second merge occurs at different locations. By seeing this trend further, we have carried out study for higher values of mass ratio as shown in Figure 19. It is noticed that as the mass ratio increases, second merge near 1:1 resonance periodic orbit moves towards the more massive primary.

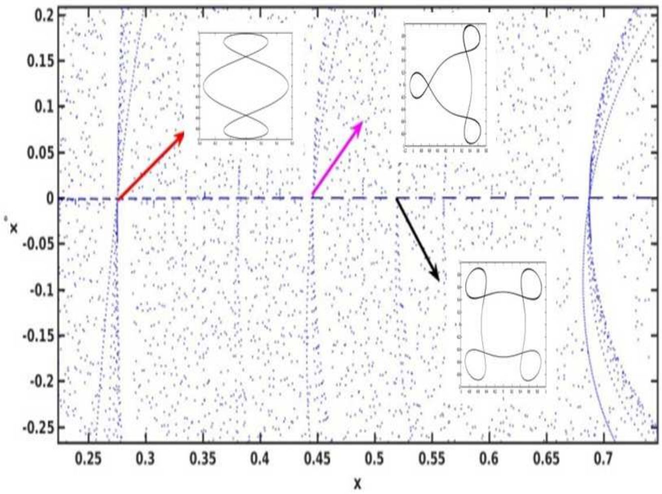

Figure 16: PSS for Jacobi Constant C=2.9 and ε=0 in the Sun-Mercury System

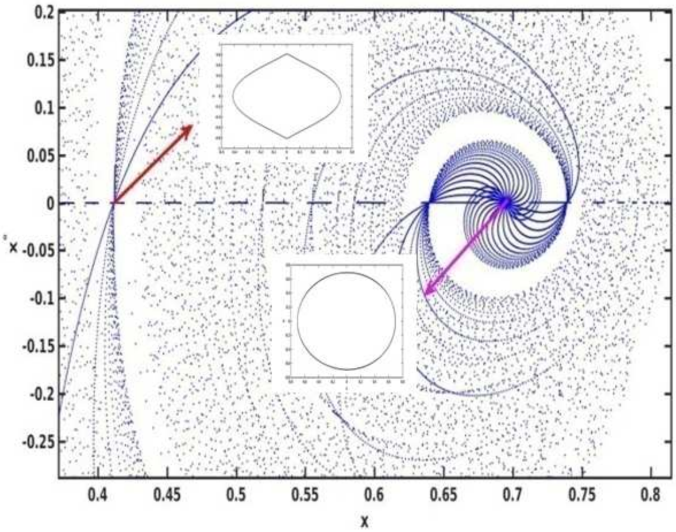

Figure 17: PSS for Jacobi Constant C=2.9 and ε= 0.09

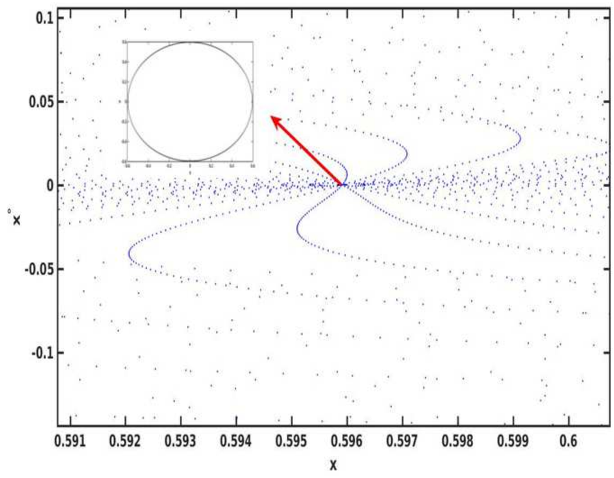

Figure 18: PSS for Jacobi Constant C=2.9 and ε= 0.13

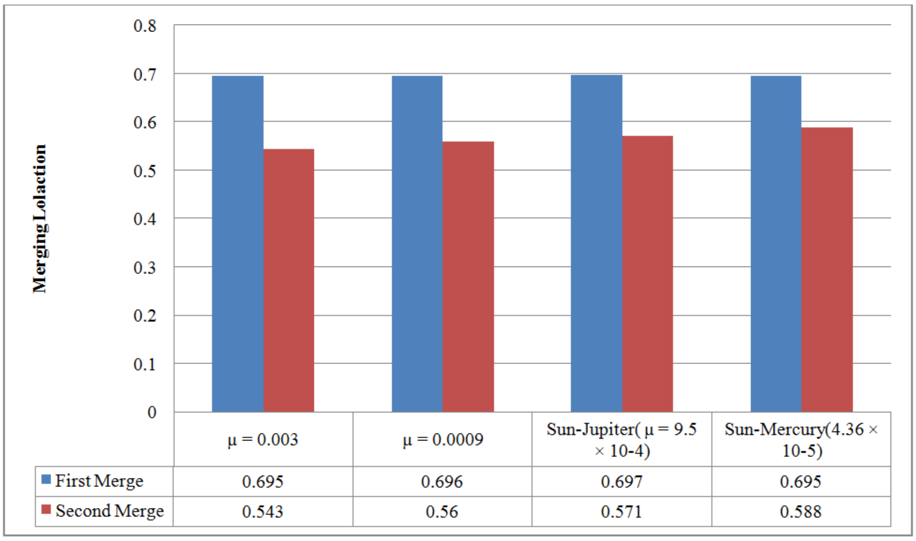

Figure 19

7. PERIODIC ORBIT AROUND JUPITER

PSS technique reveled the Jupiter centered periodic orbit at x = 0.6925 for C = 2.9, which is shown in Figure 20. This orbit around Jupiter is initially elliptical shaped elongated towards Jupiter, and gradually becomes circular with the increase in radiation pressure. As the value of radiation pressure increases, this orbit moves towards the Jupiter. The period and size of orbits decrease with increase in Sun's radiation pressure. It may be seen from the Figure 21 and Figure 23 that, the nature of variation in the orbit is different with change in radiation pressure.

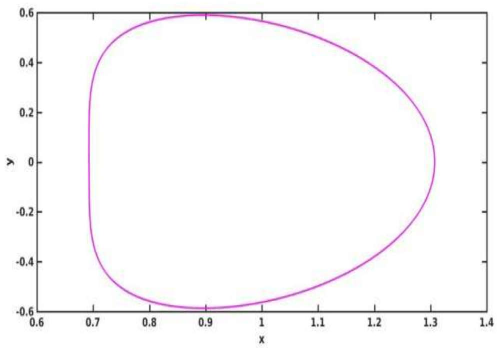

Figure 20: C = 2.9, q =1, x= 0.6925

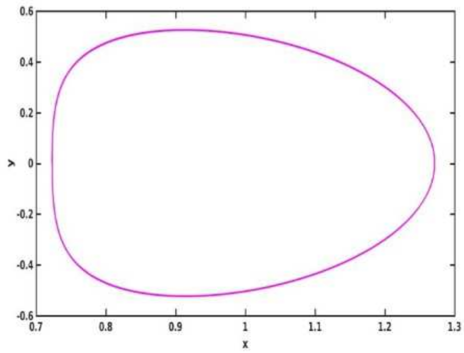

Figure 21: C = 2.9, q =0.99, x= 0.7726

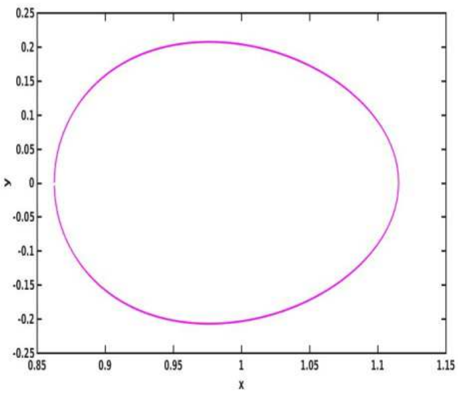

Figure 22: C = 2.9, q =0.96, x= 0.8627

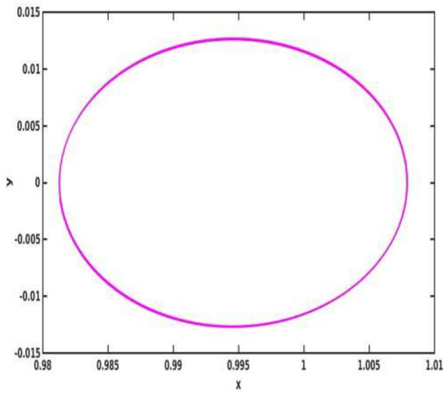

Figure 23: C = 2.9, q =0.92, x= 0.9813.

8. CONCLUSIONS

- We have studied the effect of radiation pressure in Sun-Jupiter and Sun-Mercury systems within the frame work of RTBP, using PSS metod. Interior mean motion resonances located and the effect of radiation pressure was studied on the location and size of periodic orbits.
- The merging of resonant periodic orbits with 4:3, 3:2 and 2:1 first-order interior resonances into 1:1 resonance around the Sun in the Sun-Jupiter and Sun- Mercury systems are found with the help of PSS.
- The solar radiation pressure plays key role in generating these merging.
- The radiation pressure decreases with the increase in Jacobian constant for merging. At the time of merging, these orbits become near-circular. The period and size of these orbits reduce with increase in radiation pressure.
- It is found that the elongated elliptic periodic orbit around Jupiter is changed to circular periodic orbit around it, due to the effect of radiation pressure. The present study could be helpful for the future mission design.

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